Cambridge IGCSE

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER				

MATHEMATICS **0580/41**

Paper 4 (Extended) **May/June 2016**

2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator
Tracing paper (optional).
Geometrical instruments

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
If working is needed for any question it must be shown below that question.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.

The syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 1/Level 2 Certificate.

This document consists of **19** printed pages and **1** blank page.

© UCLES 2016

CAMBRIDGE International Examinations

1 **(a)** Kristian and Stephanie share some money in the ratio 3 : 2.
Kristian receives $72.

View solution

(i) Work out how much Stephanie receives.

$ ..... [2]

(ii) Kristian spends 45% of his $72 on a computer game.

View solution

Calculate the price of the computer game.

$ ..... [1]

(iii) Kristian also buys a meal for $8.40 .

Calculate the fraction of the $72 Kristian has left after buying the computer game and the meal.
Give your answer in its lowest terms.

View solution

..... [2]

(iv) Stephanie buys a book in a sale for $19.20 .
This sale price is after a reduction of 20%.

View solution

Calculate the original price of the book.

$ ..... [3]

© UCLES 2016

(b) Boris invests \$550 at a rate of 2% per year simple interest.

View solution

Calculate the amount Boris has after 10 years.

\$ ..... [3]

(c) Marlene invests \$550 at a rate of 1.9% per year compound interest.

View solution

Calculate the amount Marlene has after 10 years.

\$ ..... [2]

(d) Hans invests \$550 at a rate of x% per year compound interest.
At the end of 10 years he has a total amount of \$638.30, correct to the nearest cent.

View solution

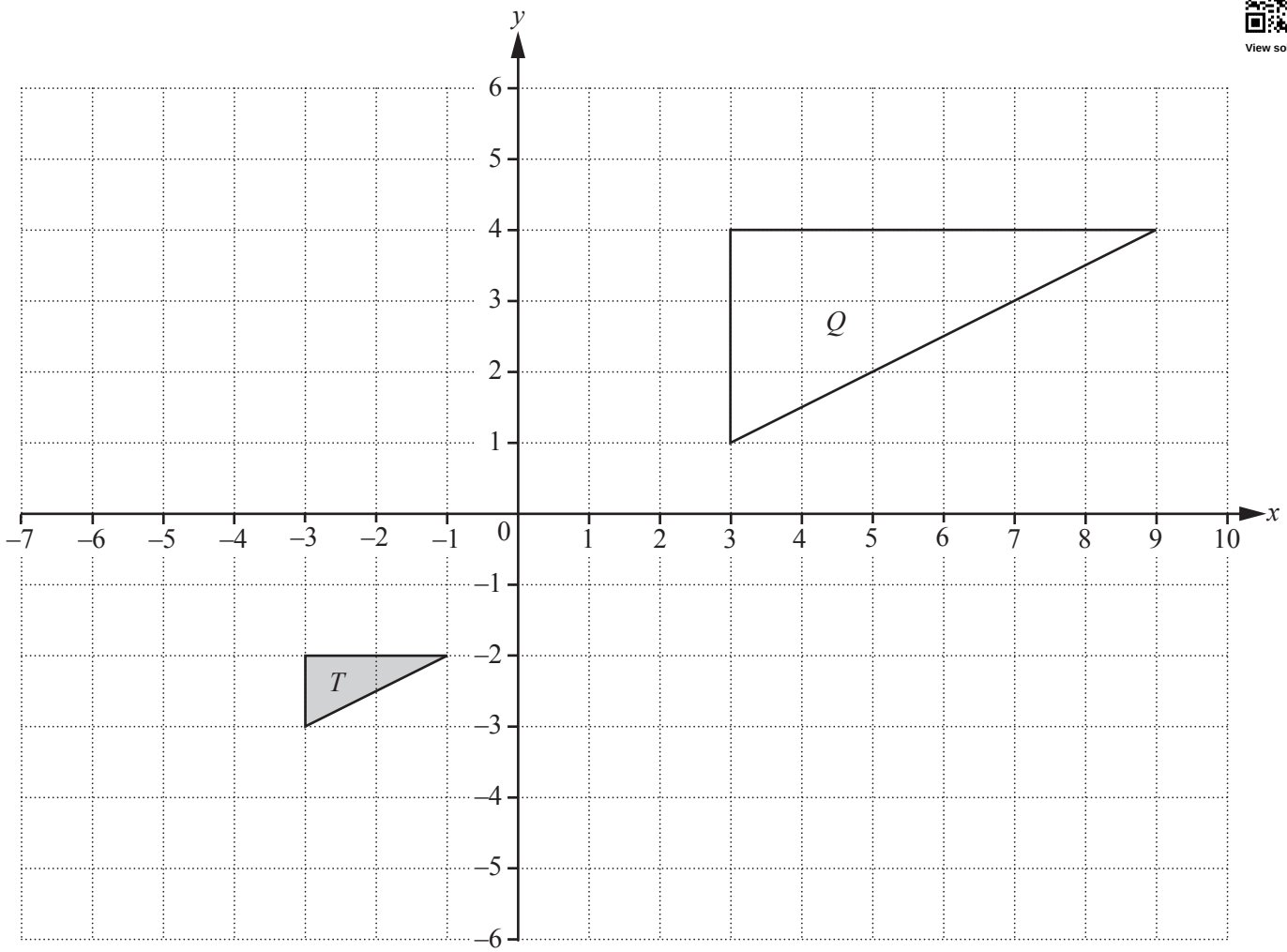
Find the value of x.

$x =$ ..... [3]

© UCLES 2016

2 (a)

View solution



(i) Draw the image of triangle T after a translation by the vector $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$. [2]

(ii) Draw the image of triangle T after a reflection in the line $y = 1$. [2]

(iii) Describe fully the **single** transformation that maps triangle T onto triangle Q.

.....

..... [3]

© UCLES 2016

(b) $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ $\mathbf{N} = \begin{pmatrix} 4 & 3 \\ 1 & k \end{pmatrix}$ $\mathbf{P} = \begin{pmatrix} 1 & 3 \\ 0 & 6 \end{pmatrix}$

View solution

(i) Work out $\mathbf{M} + \mathbf{P}$.

( ) [1]

(ii) Work out $\mathbf{PM}$.

View solution

( ) [2]

(iii) $|\mathbf{M}| = |\mathbf{N}|$

View solution

Find the value of k.

$k =$ ..... [3]

(c) **(i)** Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

View solution

.....

..... [3]

(ii) Find the matrix which represents a reflection in the line $y = x$.

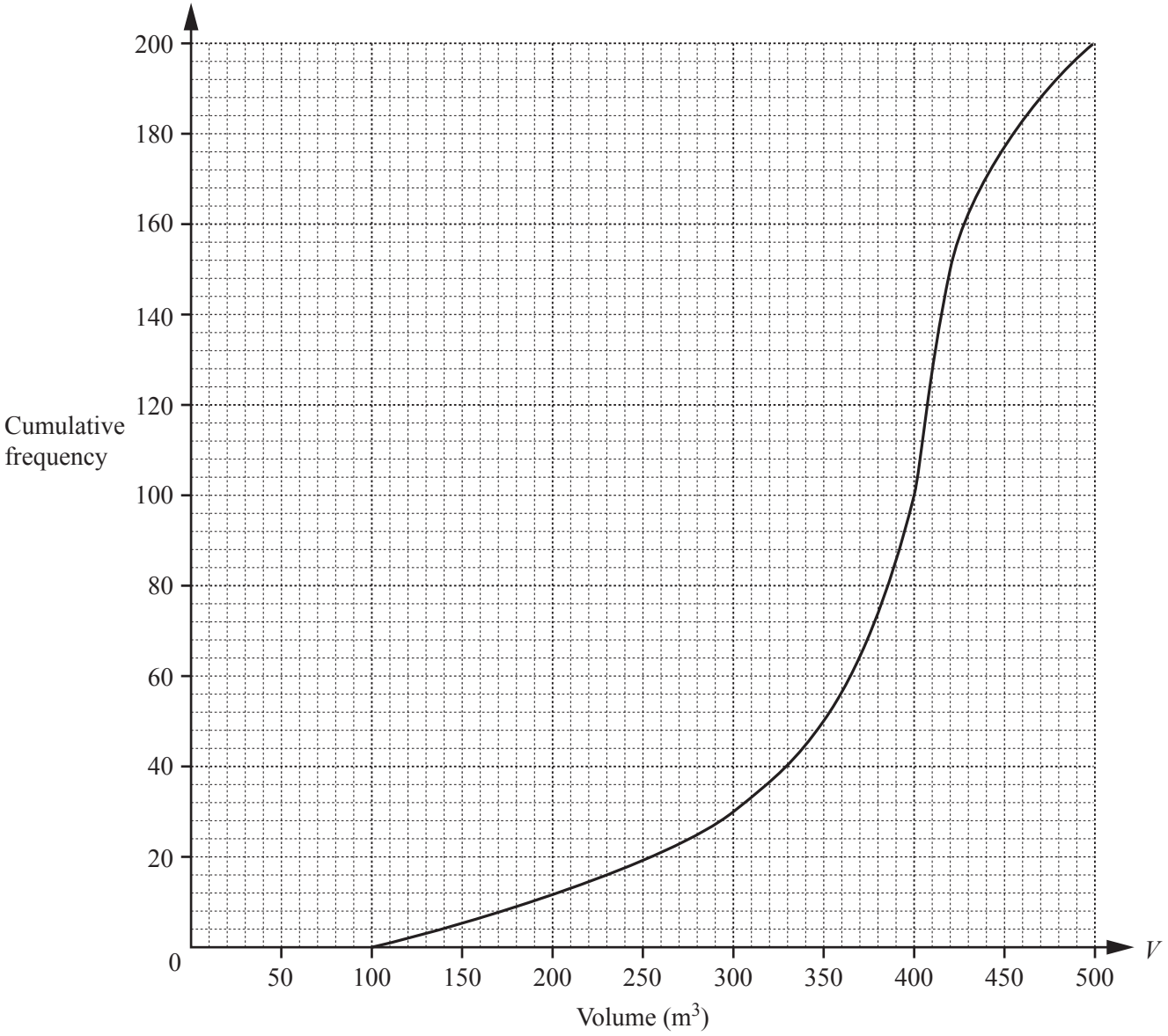
View solution

( ) [2]

© UCLES 2016

3 **(a)** 200 students estimate the volume, V m^3, of a classroom.
The cumulative frequency diagram shows their results.

View solution



Find

(i) the median,

..... m^3 [1]

(ii) the lower quartile,

..... m^3 [1]

(iii) the inter-quartile range,

..... m^3 [1]

(iv) the number of students who estimate that the volume is greater than 300 m^3.

..... [2]

© UCLES 2016

(b) The 200 students also estimate the total area, $A\,m^2$, of the windows in the classroom.
The results are shown in the table.

View solution

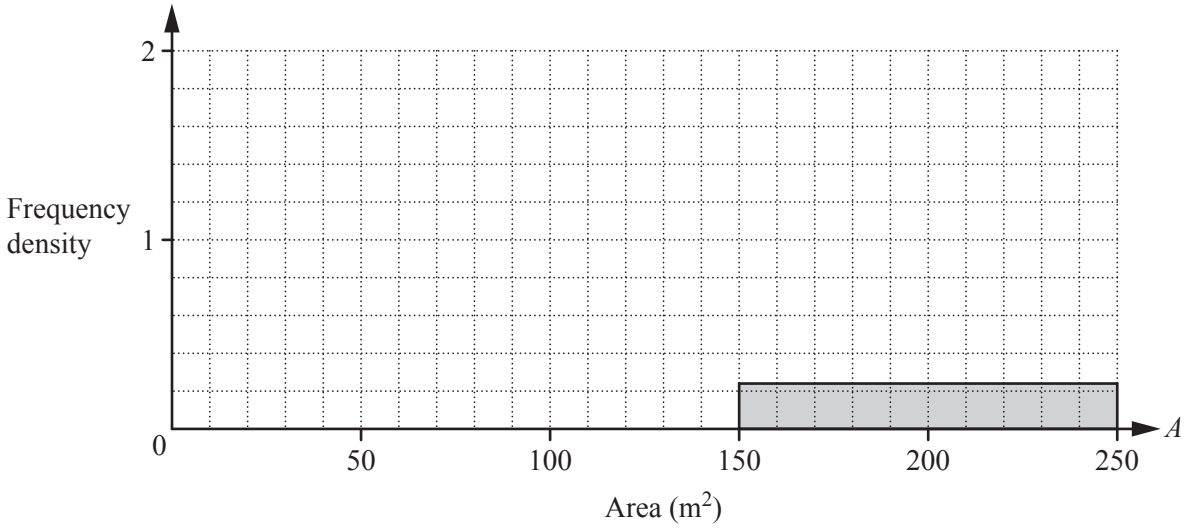
Area ($A\,m^2$)	$20 < A \leqslant 60$	$60 < A \leqslant 100$	$100 < A \leqslant 150$	$150 < A \leqslant 250$
Frequency	32	64	80	24

(i) Calculate an estimate of the mean.
Show all your working.

..... m^2 [4]

(ii) Complete the histogram to show the information in the table.

View solution



[4]

(iii) Two of the 200 students are chosen at random.

View solution

Find the probability that they both estimate that the area is greater than $100\,m^2$.

..... [2]

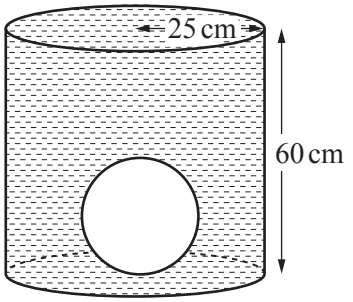
4 **(a)** Calculate the volume of a metal sphere of radius 15 cm and show that it rounds to 14 140 cm^3, correct to 4 significant figures.
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

View solution

[2]

(b) **(i)** The sphere is placed inside an empty cylindrical tank of radius 25 cm and height 60 cm. The tank is filled with water.

View solution



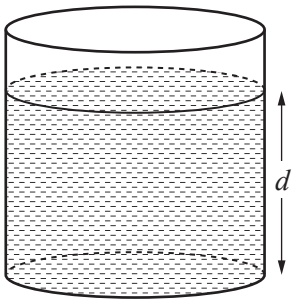
NOT TO SCALE

Calculate the volume of water required to fill the tank.

..... cm^3 [3]

(ii) The sphere is removed from the tank.

View solution



NOT TO SCALE

Calculate the depth, d, of water in the tank.

d = ... cm [2]

© UCLES 2016

(c) The sphere is melted down and the metal is made into a solid cone of height 54 cm.

View solution

(i) Calculate the radius of the cone.
[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

..... cm [3]

(ii) Calculate the **total** surface area of the cone.
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi r l$.]

View solution

..... cm^2 [4]

5 $f(x) = \frac{20}{x} + x, \quad x \neq 0$

View solution

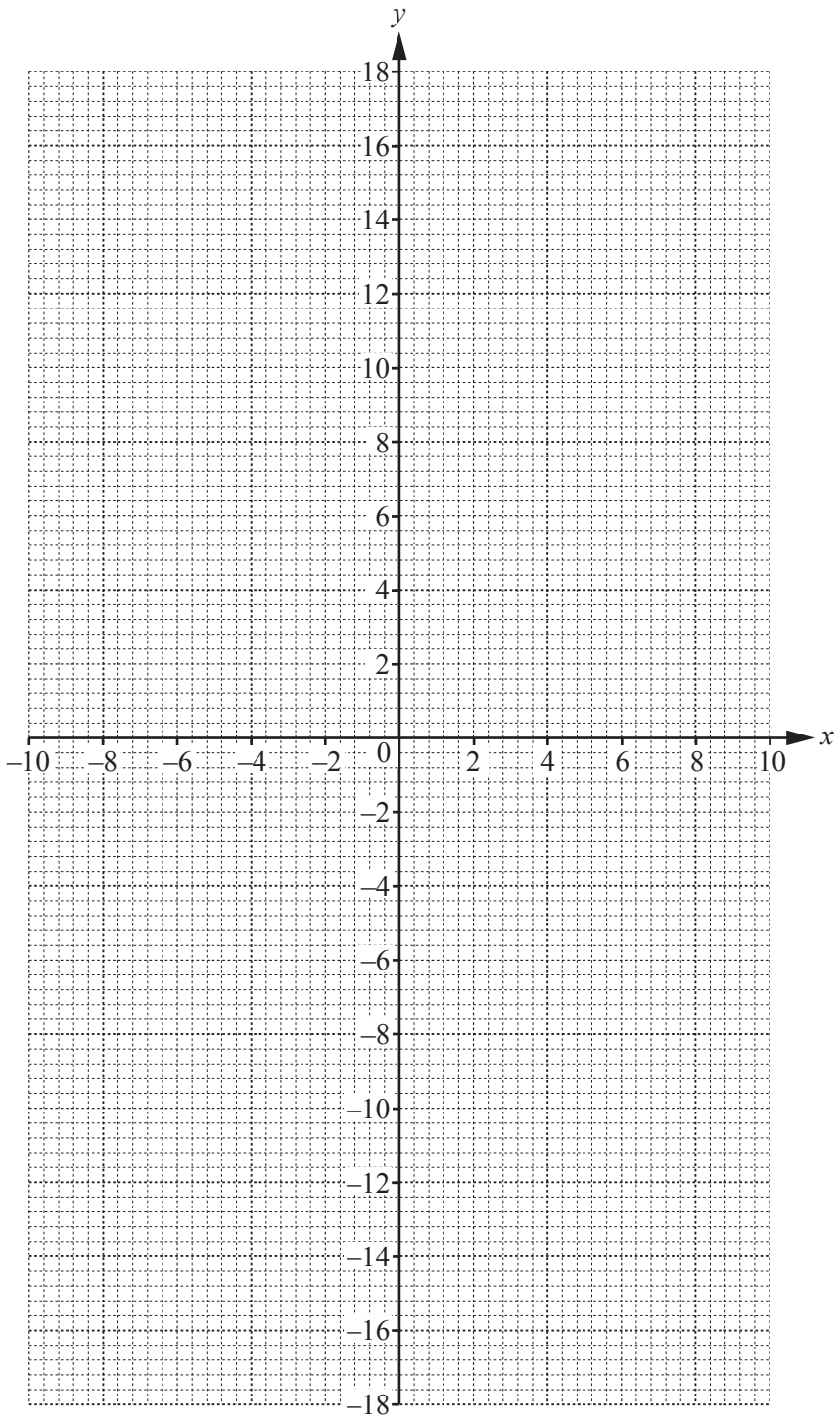
(a) Complete the table.

x	-10	-8	-5	-2	-1.6		1.6	2	5	8	10
$f(x)$	-12	-10.5	-9	-12	-14.1		14.1	12			12

[2]

View solution

(b) On the grid, draw the graph of $y = f(x)$ for $-10 \leqslant x \leqslant -1.6$ and $1.6 \leqslant x \leqslant 10$.



[5]

© UCLES 2016

(c) Using your graph, solve the equation $f(x) = 11$.

View solution

$x =$ ..... or $x =$ ..... [2]

(d) k is a prime number and $f(x) = k$ has no solutions.

View solution

Find the possible values of k.

..... [2]

(e) The gradient of the graph of $y = f(x)$ at the point (2, 12) is -4.

View solution

Write down the co-ordinates of the other point on the graph of $y = f(x)$ where the gradient is -4.

(..... , ...) [1]

(f) **(i)** The equation $f(x) = x^2$ can be written as $x^3 + px^2 + q = 0$.

View solution

Show that $p = -1$ and $q = -20$.

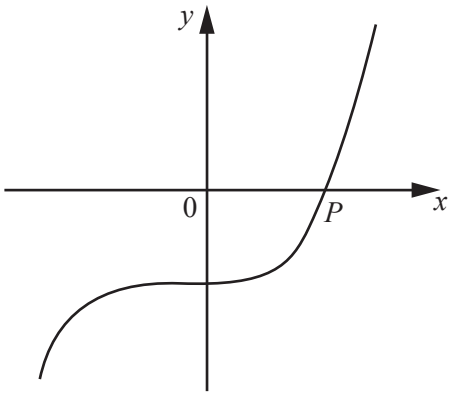
[2]

(ii) On the grid opposite, draw the graph of $y = x^2$ for $-4 \leqslant x \leqslant 4$. [2]

(iii) Using your graphs, solve the equation $x^3 - x^2 - 20 = 0$.

$x =$ ..... [1]

(iv)



NOT TO SCALE

The diagram shows a **sketch** of the graph of $y = x^3 - x^2 - 20$.
P is the point $(n, 0)$.

Write down the value of n.

$n =$ ..... [1]

© UCLES 2016

6 (a)



NOT TO SCALE

The perimeter of the rectangle is 80 cm.
The area of the rectangle is A cm^2.

(i) Show that $x^2 - 40x + A = 0$.

[3]

(ii) When $A = 300$, solve, by factorising, the equation $x^2 - 40x + A = 0$.

$x =$ ..... or $x =$ .....[3]

(iii) When $A = 200$, solve, by using the quadratic formula, the equation $x^2 - 40x + A = 0$.
Show all your working and give your answers correct to 2 decimal places.

$x =$ ..... or $x =$ .....[4]

© UCLES 2016

(b) A car completes a 200 km journey with an average speed of x km/h.
The car completes the return journey of 200 km with an average speed of $(x + 10)$ km/h.

View solution

(i) Show that the difference between the time taken for each of the two journeys is $\frac{2000}{x(x+10)}$ hours.

[3]

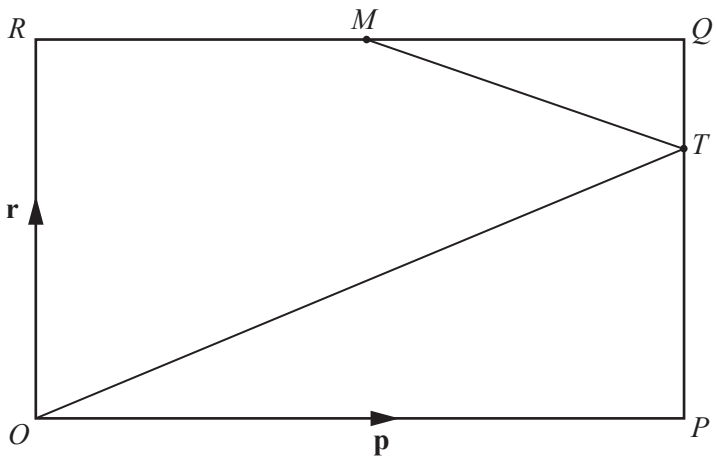
(ii) Find the difference between the time taken for each of the two journeys when $x = 80$.
Give your answer in **minutes** and **seconds**.

View solution

..... min ..... s [3]

© UCLES 2016

7



NOT TO SCALE

$OPQR$ is a rectangle and O is the origin.
M is the midpoint of RQ and $PT : TQ = 2 : 1$.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$.

(a) Find, in terms of **p** and/or **r**, in its simplest form

View solution

(i) $\overrightarrow{MQ}$,

$\overrightarrow{MQ} =$ ..... [1]

(ii) $\overrightarrow{MT}$,

View solution

$\overrightarrow{MT} =$ ..... [1]

(iii) $\overrightarrow{OT}$.

View solution

$\overrightarrow{OT} =$ ..... [1]

(b) RQ and OT are extended to meet at U.

View solution

Find the position vector of U in terms of **p** and **r**.
Give your answer in its simplest form.

..... [2]

© UCLES 2016

(c) $\overrightarrow{MT} = \begin{pmatrix} 2k \\ -k \end{pmatrix}$ and $|\overrightarrow{MT}| = \sqrt{180}$.

View solution

Find the positive value of k.

$k =$ ..... [3]

8 $f(x) = 2x + 1$ $g(x) = x^2 + 4$ $h(x) = 2^x$

(a) Solve the equation $f(x) = g(1)$.

View solution

$x =$ ..... [2]

(b) Find the value of $fh(3)$.

View solution

..... [2]

(c) Find $f^{-1}(x)$.

View solution

$f^{-1}(x) =$ ..... [2]

(d) Find $gf(x)$ in its simplest form.

View solution

..... [3]

© UCLES 2016

(e) Solve the equation $h^{-1}(x) = 0.5$.

View solution

$x =$ ..... [1]

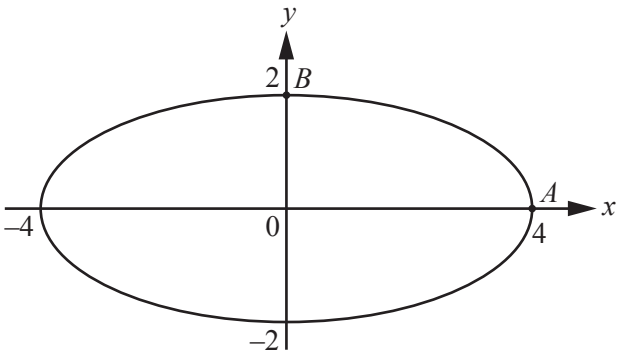
(f) $\dfrac{1}{h(x)} = 2^{kx}$

View solution

Write down the value of k.

$k =$ ..... [1]

9



NOT TO SCALE

The diagram shows a curve with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(a) A is the point (4, 0) and B is the point (0, 2).

View solution

(i) Find the equation of the straight line that passes through A and B.
Give your answer in the form $y = mx + c$.

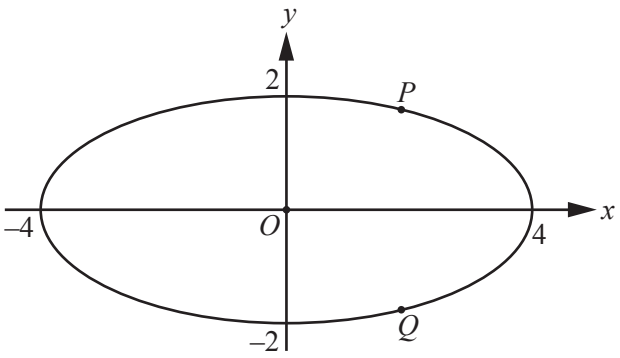
$y =$ ..... [3]

(ii) Show that $a^2 = 16$ and $b^2 = 4$.

View solution

[2]

(b)



NOT TO SCALE

$P\,(2, k)$ and $Q\,(2, -k)$ are points on the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

(i) Find the value of k.

$k =$ ..... [3]

(ii) Calculate angle POQ.

Angle $POQ =$ ..... [3]

(c) The area enclosed by a curve with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab.

(i) Find the area enclosed by the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

Give your answer as a multiple of π.

..... [1]

(ii) A curve, mathematically similar to the one in the diagrams, intersects the x-axis at (12, 0) and (−12, 0).

Work out the area enclosed by this curve, giving your answer as a multiple of π.

..... [2]

© UCLES 2016

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge International Examinations Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cie.org.uk after the live examination series.

Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

© UCLES 2016